# Hydropower Economics : An Overview

**by**

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## 1. Introduction

Electricity produced by hydropower is an important source of renewable energy. About 20 % of the electricity in the world is generated by hydropower. About one third of all countries in the world depend on hydropower for over 50% of their electricity generation (in 2001). The 10 largest producers of hydroelectricity (in 2009) are presented in Table 1, where output, capacity

*Table 1. Ten of the largest hydroelectric producers as at 2009*

| Country | Annual hydroelectric production (TWh) | Installed capacity (GWh) | % of total capacity |
|---|---|---|---|
| China | 652 | 197 | 22 |
| Canada | 370 | 89 | 61 |
| Brazil | 364 | 69 | 86 |
| United States | 251 | 80 | 6 |
| Russia | 167 | 45 | 18 |
| Norway | 141 | 28 | 98 |
| India | 116 | 34 | 16 |
| Venezuela | 86 | 15 | 69 |
| Japan | 69 | 27 | 7 |
| Sweden | 66 | 16 | 44 |

*Source: Wikipedia: electricity*

and share of electricity production is given. The largest producer, China, also has the single largest production unit; the Three Gorge Dam completed in 2008. Hydro represents only 22 % of electricity output of China. The smallest contribution of hydro is in USA where hydroelectricity only accounts for 6 % of total electricity production, while it counts for 98 % of the electricity production of Norway that is the sixth largest hydropower producer. The different ratios of capacity and energy production between countries indicate different use of hydroplanes as base load and peak load capacity, and differences in the relative availability of inflows.

*Environmental problems*

Hydropower is often termed green energy because its production does not generate harmful emissions. However, the main environmental problem is the exploitation of hydropower sites as such. Reservoirs are often artificially created, flooding former natural environments or inhabited areas, although in Norway many reservoirs are based on natural lakes in remote mountain areas. Furthermore, water is drained from lakes and watercourses and transferred through tunnels over large distances, and finally there are the pipelines from the reservoir to the turbines that often are visible, but they may also go inside mountains in tunnels. Thus hydropower systems "consume" the natural environment itself. The waterfalls, lakes and rivers that visitors enjoyed are not there anymore.

There may also be current environmental problems due to the change in the reservoir level and the amount of water downstream. Freshwater fish like trout and salmon may suffer, and is especially vulnerable in the spawning season. Changing reservoir levels may create problems for aquatic life with implications for fish, as may also changing levels of release of water downstream, in addition to problems for agriculture in changing the microclimate in the areas of the previously natural rivers and streams. In countries where substantial plant- or tree-covers are flooded when constructing a dam there may be creation of methane gases when plants are rotting in the water, either released directly or when the water is processed in turbines. During dry seasons the banks of the dam may re-grow with plants, so the rotting process is continuing once the dam is filled again.

*Fundamental physics of hydropower*

Hydropower is based on water driving the turbines generating electricity by induction. The primary energy is provided by gravity and the height the water falls down on to the turbine. Hydropower can be based on unregulated river flows, or dams with limited storage capacity above the natural flow, and on water drawn from reservoirs that may contain up to several years worth of inflow. A reservoir is the key to the economic utilization of hydropower when there is a difference between the pattern of consumption of electricity and the seasonal inflows filling up the reservoir. Figure 1 shows a typical year for the situation in Norway concerning production

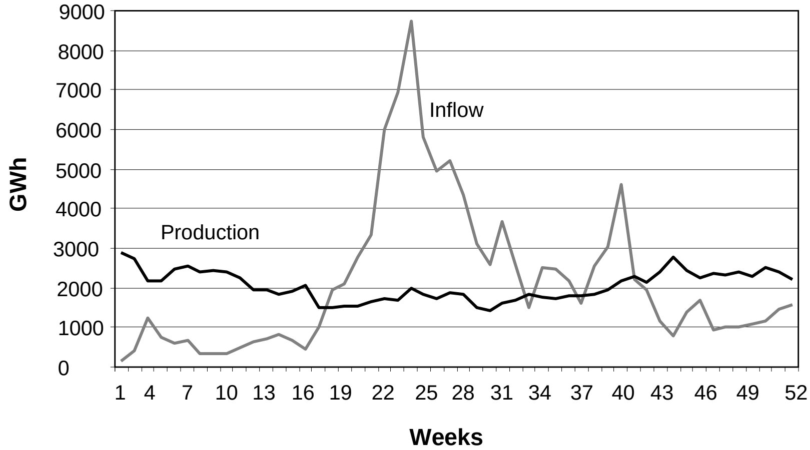


*Figure 1. Weekly inflow and production of hydropower in Norway 2003*
*Source: OED: Fakta 2005*

and inflows. The role of the dams is to permit a transfer of water from the late spring, summer, and early autumn weeks to the late autumn and winter weeks. In Norway as in other countries with snow in the winter the reservoirs are filled up when the snow melts in spring and summer.

The transformation of water into electricity for a plant with a reservoir can be captured in the simplest way by the production function for a hydro plant

$$e_t^H = \frac{1}{a} r_t \,, \tag{1}$$

where $e_t^H$ is the energy (in kWh) produced during a period, $r_t$ is the water released onto the turbine during the period, and $a$ is the input coefficient or unit requirement of water; showing how many units of water (e.g. cubic meters) are required to produce 1 kWh of electricity. If the power station does not have a reservoir, i.e., if it is based on a river flow, then this flow is substituted for the release of water. For an efficiently run operation we will have equality in the production relation (1) above.

The potential for electricity of one unit of water is associated with the height from the dam level to the turbine level. The reservoir level will change somewhat when water is released and thus influence electricity production. Electricity production is also influenced by how water is transported away from the turbine, allowing new water to enter. The turbine is constructed for an optimal flow of water. Lower or higher inflow of water may reduce electricity output per unit of water somewhat. Variation of energy conversion efficiency according to utilization of a turbine; ranging for 80% for a low utilization to 95-96% maximally, and then a reduction again if more water is let on to the turbine.

Neither real capital nor other current inputs like labour and materials are entered in the production function. The role of capital is to provide a capacity to produce electricity; therefore is can be suppressed in an analysis of managing the given capacity. Technology is typically embodied in the capital structure. The fabrication coefficient will reflect the embodied technology of feeding tunnels and turbines, and the engineering design of optimal water release on to the turbines. We will disregard detailed engineering information about energy efficiency.

The nature of the costs is important for optimal management of current operations. Given that capacities are present and fixed, only variable costs should influence current operations. However, our specification does not show any input other than water, and the water is not bought on a market. Empirical information indicates that traditional variable costs, i.e., costs that vary with the level of output, can be neglected as insignificant. People are employed to overlook the processes and will be there in the same numbers although the output may fluctuate. Maintenance is mainly a function of size of capital structure and not the current output level. (However, wear and tear of turbines depends on the number of start-ups.) We will therefore assume that there are zero current costs. This is a very realistic assumption for hydropower. Water represents the only variable cost in the form of an *opportunity cost* as mentioned above, i.e., the cost today is the benefit obtained by using water tomorrow.

The reduced electricity conversion efficiency due to a reduced height the water falls as the reservoir is drawn down is disregarded in the production function (1). For the Norwegian system, with relatively few river stations and high differences in elevations between dams and turbine stations of most of the dams (the average height is above 200 meters), this is an acceptable simplification at our level of aggregation. In more technically-oriented analyses it may be specified that the coefficient varies with the utilization of the reservoir. The production function is extremely simple. Topology and the constructed wall of the dam give the height so it may be included in the calculation of the input coefficient.

There are two distinct decision problems concerning hydropower: the investment decision in new capacities and the management decision of operating existing capacities. The chapter will only be concerned with the latter problem. The chapter will focus on some unique features of hydroelectricity management problems in detail, and then mention other interesting topics very briefly. Main conclusions are illustrated using innovative bathtub diagrams. There is a huge engineering literature in the form of journal papers, but a relatively sparse literature based on economics..

## 2. The basic model for management of hydropower

*Hydropower with a reservoir*

A most simple model for managing a hydropower system will be specified. We are going to use discrete time. This is the case for all practical applications of the type of model we are analysing. The variables are going to be of two types, flow and stock. Stock variables must be dated, e.g., either at the start or at the end of a period. The flow variables will be interpreted as magnitudes related to realisation during a period.

The dynamics of water management is based on the filling and emptying of the reservoir. These activities are modeled by the water accumulation equation

$$R_t \leq R_{t-1} + w_t - r_t ,\ t = 1,..,T \qquad (2')$$

$T$ is the number of periods we are considering, $R_t$ is the amount of water in the reservoir at the end of period $t$, $w_t$ is the inflow to the reservoir, and $r_t$ is the release of water onto the turbines, as defined above in connection with the production function. The amount of water in the reservoir at the end of period $t$ is less than or equal to the amount of water received from the previous period $t$ -1, plus the inflow during the period and subtracted the use of water for electricity production during the period. Evaporation from the reservoir is not accounted for explicitly. This is quite reasonable for a northern country like Norway, but may be dealt with in the definition of inflow for warmer countries. A reservoir has a limited size

$$R_t \leq \bar{R} \tag{3}$$

$\bar{R}$ is the maximal amount the reservoir can room, so overflow may happen. In that case we have a strict inequality in Equation (2') and $R_t = \bar{R}$. For simplicity the lower limit is set to zero, so $\bar{R}$ has the interpretation of the maximal amount of water that can be utilised. (If a study of the lower limit is explicitly needed, e.g. due to a revision of the minimum amount, this can easily be accommodated.)

The production function (1) can be substituted into the water accumulation Equation (2') yielding

$$\begin{aligned} &R_t \leq R_{t-1} + w_t - r_t = R_{t-1} + w_t - ae_t^H \Rightarrow \\ &\frac{R_t}{a} \leq \frac{R_{t-1}}{a} + \frac{w_t}{a} - e_t^H \end{aligned} \tag{4}$$

In order to simplify the notation we will measure all variables in the energy unit kWh and re-write the water accumulation as

$$R_t \leq R_{t-1} + w_t - e_t^H \text{ , } t = 1,..,T \tag{2}$$

All variables are now measured in kWh, but we will still continue to talk about use of water, etc., since this has a more intuitive appeal.

*The social planning problem*

We need to formulate an objective function for the problem of managing the capacities of the system over time. To repeat, our problem is dynamic because of the existence of reservoirs we have a choice of which period to use the water, today or tomorrow. An engineering approach to this type of problem is typically to minimize costs for given demands for each period. However, hydropower has zero variable costs, and fixed costs that are sunk are not of interest in a management problem provided it is optimal to use the existing hydropower system. We will assume that this is the case. The simplest economic objective function is to maximize the benefit of consumers and producers from producing electricity. The objective function can then be expressed as

$$\sum_{t=1}^{T} \int_{z=0}^{e_t^H} p_t(z)\, dz \qquad (5)$$

Because there is no variable costs the benefit of electricity production, called consumer plus producer surplus, is measured by the area under the demand functions $p_t = p_t(e_t^H)$ (written on price form) for each period.

The price concept in the analysis will be of the nature of socially optimal prices. Whether such prices coincide with market prices, or prices that consumers actually pay, is another (very interesting) question. A price measures the consumers' marginal willingness to pay, so we can say that the objective function (5) measured in a money unit, represents a type of social welfare function disregarding all distributional issues.

We will regard the issue of optimal management of the hydro capacities in isolation, disregarding any links with the rest of the economy. Our problem is therefore a partial problem that is an approximation to a more general analysis of the role of electricity in a real economy. The social optimization problem using as the objective function the total sum of consumer plus producer surplus, taking into account the water accumulation function (2) and the limited reservoir (4), is then

$$\begin{aligned}
&\max \sum_{t=1}^{T} \int_{z=0}^{e_t^H} p_t(z)\, dz \\
&\text{subject to} \\
&R_t \le R_{t-1} + w_t - e_t^H \qquad (6)\\
&R_t \le \bar{R} \\
&R_t, e_t^H \ge 0\ ,\ t = 1,..,T \\
&T, w_t, R_o, \bar{R} \text{ given, } R_T \text{ free}
\end{aligned}$$

Two simplifications may be noted. Firstly, there is no discounting appearing in the objective function of Equation (6). The planning horizon *T* for a hydro management problem is in practice rather short, from 1 to 5 years. Any discounting with a real rate of interest of magnitude 4-6 % will then have very modest impact on the results. Secondly, the amount of water left in the reservoir at the end of the planning horizon is free, typically implying that nothing will be left. (Both these features are easy to change in the model formulation.)

The optimisation problem (6) is a discrete time dynamic programming problem, and special solution procedures have been developed for this class of problems. However, we shall not go into technical details here, but try to get an understanding of some fundamental qualitative results using direct reasoning first, and then introduce graphical illustrations.

Let us first make some reasonable economic assumptions. We are dealing with a system-wide model, so an assumption of strictly positive amounts of electricity produced in each period seems obvious. Furthermore, it is also reasonable to assume that the social prices will be strictly positive for all periods. Due to the fact that electricity is a perfect homogeneous good and that we have not introduced any time preferences (no discount factor) the social solution cannot be better than if it is possible to follow perfect arbitrage and have the same price in every period. Consider the case that the price for a period is higher than for all other periods having a common price. The marginal value of electricity is then higher in the period with the higher price than in any of the other periods. This means that the value of the objective function (5) can be increased by producing more electricity in this period at the expense of the other periods. The value of the objective function cannot be higher than if all prices are equal.  But the restrictions in the optimisation problem (6) may prevent such a solution to be feasible.

The general solution principle for dynamic programming problems is Bellman's backward induction. We can therefore start with solving for the last period, *T*. It follows directly that the amount of water available in this period is $R_{T-1} + w_T$. Then, due to our assumption of no restriction on the amount of water to leave at the end of the planning period we have that all the available water will be produced, yielding

$$e_T^H = R_{T-1} + w_T \quad (7)$$

The social price becomes

$$p_T = p_T(e_T^H) = p_T(R_{T-1} + w_T) \quad (8)$$

We note that the optimal amount of electricity and the price are conditional on the amount of water transferred from period *T*-1 to period *T*. The range of this transfer is

$$R_{T-1} \in [0, \bar{R}] \quad (9)$$

Thus, we may have two corner solutions; when the minimal amount of zero is transferred, i.e., all available water is used in period *T*-1 leaving the reservoir empty, and when the maximal amount $\bar{R}$ is left to the next period. For an interior solution the amount left to the next period is in between these limits. A fundamental insight of optimal pricing follows from these basic possibilities: an optimal price will only change if one of the constraints on the reservoir is binding. This was stated already by the Norwegian electricity regulator (the Norwegian Water Resources and Energy Directorate) in 1968. The arbitrage reasoning tells us that as long as we have an interior solution the price remains the same.

Stepping one period back in time to *T*-1 we have that the amount of water received from the previous period *T*-2 is $R_{T-2}$ and the optimal amount to leave to period *T* is $R_{T-1}$, leaving the optimal amount of electricity to be

$$e_{T-1}^H = R_{T-2} - R_{T-1} + w_{T-1} \quad (10)$$

Now assuming an interior solution for the amount $R_{T-1}$ to leave for period *T* we know that the price in period *T*-1 will be the same as in period *T*, implying

$$p_{T-1}(e_{T-1}^H) = p_T(R_{T-1} + w_T) \Rightarrow e_{T-1}^H = p_{T-1}^{-1}(p_T(R_{T-1} + w_T)) \quad (11)$$

Using (10) yields

$$R_{T-2} = R_{T-1} - w_{T-1} + e_{T-1}^H = R_{T-1} - w_{T-1} + p_{T-1}^{-1}(p_T(R_{T-1} + w_T)), \quad (12)$$

where we have substituted from Equation (11) to obtain the last expression.

Moving on to period *T*-2 we have

$$R_{T-3} = R_{T-2} - w_{T-2} + e^H_{T-2} = R_{T-2} - w_{T-2} + p^{-1}_{T-2}(p_T(R_{T-1} + w_T)) \tag{13}$$

We now see that repeating this reasoning step after step right to period 1 backwards in time, keeping the assumption that all optimal transfers of water from one period to the next are interior solutions, implying that all the prices are the same, we have that by successively inserting the solution for the reservoir level starting with $R_{T-2}$ in Equation (12), we obtain at last

$$R_0 = R_{T-1} - \sum_{i=1}^{T-1} w_i + \sum_{i=1}^{T-1} p_i^{-1}(p_T(R_{T-1} + w_T)) \tag{14}$$

In this equation the only unknown is the final amount of water $R_{T-1}$ to be left to the terminal period *T*. Once we have solved for this amount using Equation (14) all the other solutions for the common price and amounts of electricity for each period follow from Equations (7), (8), (12), etc.

But we have to consider the possibilities of corner solutions. Assume that going backwards on the time axis the first period we encounter with a corner solution for the amount of water left to the next period is *t*+1 when it may be optimal to empty the reservoir, i.e. $R_{t+1} = 0$. Using Equation (14) we then have

$$R_{t+1} = R_{T-1} - \sum_{i=t+2}^{T-1} w_i + \sum_{i=t+2}^{T-1} p_i^{-1}(p_T(R_{T-1} + w_T)) \Rightarrow R_{T-1} = \sum_{i=t+2}^{T-1} w_i - \sum_{i=t+2}^{T-1} p_i^{-1}(p_T(R_{T-1} + w_T)) \tag{15}$$

The optimal price in period *t*+1 must typically be higher that the price that was constant from *T* to *t*+2; $p_{t+1} \geq p_T$, because otherwise it would not be optimal to empty the reservoir in period *t*+1.

Assuming that we have the other corner solution of leaving a full reservoir $\bar{R}$ in period *s* to period *s*+1 with *s*+1 < *t* , but have interior solutions for periods backwards from *t* to *s*+1, implying that the price is constant on this time interval, we can now use period *t*+1 in the role of the terminal period and proceed as above to obtain

$$R_s = R_t - \sum_{i=s+1}^{t} w_i + \sum_{i=s+1}^{t} p_i^{-1}(p_{t+1}(R_t + w_{t+1})) \Rightarrow R_t = \overline{R} + \sum_{i=s+1}^{t} w_i - \sum_{i=s+1}^{t} p_i^{-1}(p_{t+1}(R_t + w_{t+1})) \qquad (16)$$

Assuming that from period *s*-1 backward to the first period we again have interior solutions for all periods we will enter a new price regime again with typically a lower price than obtained for periods from *t*+1, $p_u \leq p_u + 1 = p_t + 1 \; (u = 1,..,s)$, because leaving a full reservoir representing a threat of overflow will not be optimal unless the price in the next period is higher. The level of the new and final (going backwards) price regime is found in the same way as above by solving for $R_{s-1}$ from

$$R_0 = R_{s-1} - \sum_{i=1}^{s-1} w_i + \sum_{i=1}^{s-1} p_i^{-1}(p_s(R_{s-1} + w_s)) \qquad (17)$$

We have demonstrated the price changes due to the basic reservoir constraints becoming active. There may be several price changes during a yearly cycle, depending on demand and inflow conditions. But it is rather obvious that the variation we see in prices in hydro-dominated courtiers like Norway must have additional explanatory factors.

*The bathtub diagram for two periods*

Observing the recursive structure of the optimization problem (6) in the equation giving rise to dynamics, the water accumulation Equation (2), only variables from two consecutive periods appear. This means that a sequence of two-period diagrams may capture the main features of the general solution. As we have explored above there are three conditions for price-setting regimes, the reservoir remaining within its limits of empty and full, the reservoir running empty and the reservoir running full. Focusing just on two periods is enough to bring this out. The solution for two periods can be illustrated in a bathtub diagram, Figure 2, showing the total available water

<Figure 2 in here>

for the two periods as the floor of the bathtub, and the demand curves anchored on each wall. The maximal storage is now introduced. Inflow plus the initial water $R_o$ in period 1 is *AC*, and inflow in period 2 is *CD*. The maximal storage is *BC*. The storage is measured from *C* toward the axis for period 1 because the decision of how much water to transfer to period 2 is made in

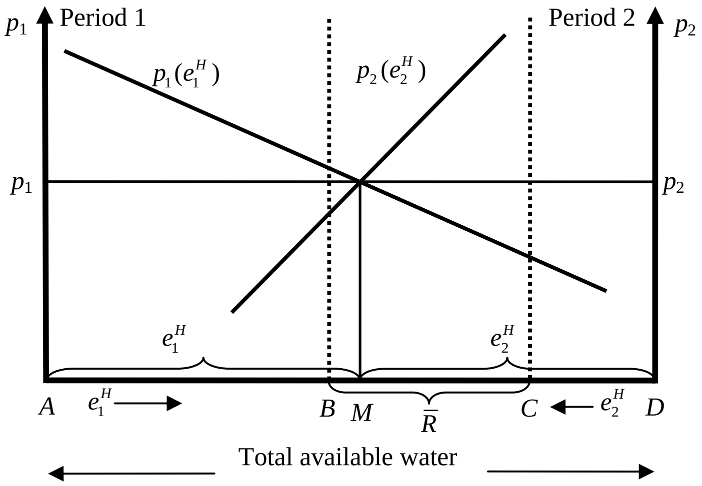


*Figure 2. Two-period bathtub diagram with non-binding reservoir constraints*

period 1. The intersection of the demand curves determines the common price for the two periods. The point *M* on the bathtub floor shows the distribution of electricity production on the two periods. The optimal transfer illustrates the case when the reservoir limit is not reached, but there is scarcity in period 2 since all available water, *MC* + *CD*, in that period is used up. Therefore the amount *AM* is consumed in period 1 and *MC* is saved and transferred to period 2. The total amount available for both periods is used up and gives rise to a positive price for both periods. The intersection of the demand curves takes place within the vertical lines from *B* and *C*, indicating the maximal storable amount. Since water consumed in period 1 is at the expense of potential consumption in period 2 the water values become the same and equal to the price for both periods.

The demand curves may also intersect to the left of the broken vertical reservoir capacity line from *B* as illustrated in Figure 3.

<Figure 3 in here>

The optimal allocation is now to store the maximal amount *BC* in period 1 because the water value is higher in the second period, and consume what cannot be stored, *AB*, in period 1. Due to the assumption of non-satiation of demand it cannot be optimal

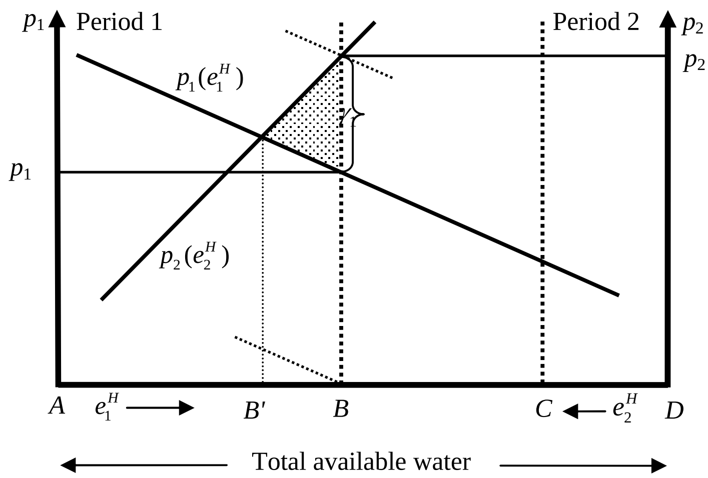


*Figure 3. Social optimum with upper reservoir constraint binding in period 1*

with any spill in period 1. The price is now higher in the second period as predicted from the discussion below (16). In the second period the reservoir, containing *BC* from the first period and an inflow of *CD* coming in the period, is emptied. We go from a period of threat of overflow to a period with scarcity. Notice that the water allocation will be the same for a wide range of period 1 demand curves keeping the same period 2- curve, or vice versa. The period 1- curve can be shifted down to passing through *B* and shifted up to passing through the level for the period 2 water value, as indicated by the dotted lines as alternative demand curves. The price difference between the periods may correspondingly vary considerably. A binding reservoir constraint implies that the value of the objective function becomes smaller. Using the unconstrained solution as a benchmark, indicated by the vertical dotted line from *B′* to the intersection of the demand curves, the marked triangle is the reduction in total consumer plus producer surplus due to the limited size of the reservoir.

The bathtub diagram may be used for just two periods as in Figures 2 and 3, but it may also be used within a multiperiod analysis for two consecutive periods. The two-period nature of the dynamics of the system makes it possible to illustrate a sequence of optimal solutions using two-period bathtub diagrams.

*Introducing more constraints*

In order to increase the realism of our model we should introduce an upper limit on how much electricity that can be produced during a period. A hydro plant has an installed power capacity that used maximally gives a constraint on the possible amount of energy during a period:

$$e_t^H \le \overline{e}^H \ , \ t = 1,..,T \tag{18}$$

The transmission system connecting producers and consumers may also limit the amount of electricity that the system can sustain. It is often the transmission system that is the limiting factor. We can let Equation (18) also represent this event since we are modelling at a system level. In principle the constraint can be taken care of in the same way as dealing with the reservoir constrain in the previous subsection. It is rather straightforward to show that a binding constraint on production shifts the price upwards.

Abrupt changes in use of water may have negative environmental effects or affect other uses of dams or rivers. Therefore constraints on ramping rates are usually imposed.

*Multiple producers*

There may be many hydropower plants in a country. In Norway there are over 700 hydropower plants, and a majority of them have reservoirs, 830 in all. It is therefore of interest to establish under what circumstances operating with an aggregated system can be of practical importance. We maintain the same assumptions as in Section 2 and regard only the constraint on the reservoirs in this section. Each plant is assigned one reservoir. A transmission system is not specified, and the plants operate independently, i.e., there are no “hydraulic couplings” as there will be between plants along the same river system. An important consequence of disregarding power, production or transmission constraints for any plant is that a plant can empty its reservoir during a single period. This can be defined as *perfect manoeuvrability* of the reservoirs. But we do not assume that inflows can be channelled to any reservoir. The inflows are reservoir or plant specific. The plants have in general different fabrication coefficients in their production functions (1), and the water-accumulation equation of the type (2) for each plant is deflated by the plant-specific fabrication coefficient, assuming no waste of water in production. The planning problem is the same as Equation (6), but now the total amount consumed is a summing up of output from each plant.

We obtain unique solutions for the *aggregate* production in any period, but not necessarily solutions for the allocation of this production on individual plants. Using the backwards-induction principle again, assuming that demand is not satiated and that all reservoirs are emptied in the terminal period *T*, due to the free terminal condition, the total consumption of electricity equal to the total production and the price common to all units, is determined in the same way as explained for the aggregated system studied above. Without overflow at any plant or any plant emptying its reservoir the price must be the same as for period *T* and common to all plants. We can go backwards to period 1 and get the same result. However, we have to check how the system price can change and what happens when there are corner solutions for individual reservoirs and plants. The system price change due to the relationship between demand and total supply, and there is no room for individual plant prices. However, the utilisation profile for individual plants does not necessarily follow uniquely from the aggregate behaviour of the system.

In a period *t* the interplay of demand and total supply determines the equilibrium piece and quantity. However, it may not matter in general for the optimal solutions which plants that contribute to total supply and how much, as long as the plants have the optimal amounts in the reservoirs when the system price changes.

The conclusion we can draw is that in periods where the social price remains constant, then the management of individual plants is not fully determined. Plants differ as to the number of periods it takes to fill the reservoir depending on the size of reservoir relative to the amount of inflow. A plant with a small storage capacity may empty the reservoir several times during a constant price regime extending many periods. But the important lesson to learn is that when there is a system price change, then all plants must have a full reservoir to transfer to the period having a price jump upwards, and if the price goes down all the plants must have empty reservoirs the period before for the solution to be optimal.

As expressed by the director of the Norwegian electricity regulator (the Norwegian Water Resources and Energy Directorate), as far as back in 1968

> …no single reservoir is overflowing before all reservoirs are filled up, and … no single reservoir is empty before all are empty.

However, if we consider the existence of plants with large reservoirs, e.g. multi-year reservoirs, and additional constraints as mentioned in Section 2 are introduced for each plant individually, then this aggregation conjecture will no longer hold exactly. Reducing the maneuverability of the reservoirs to different degrees will result in the conjecture only being valid as an approximation that will be more or less crude.

## 3. Hydro together with other technologies

It is unusual for a country to rely only on hydro for the electricity supply. It is therefore of interest to study how hydro should be utilized together with other technologies. We will consider thermal generation represented by coal-fired, gas-fired and nuclear capacities, and also renewable energy. In Europe there has been an increasing interest about the issue of combining hydro power and renewable technologies that are also intermittent. The most important forms of intermittent technologies are wind power, solar power and run-of-the river, i.e., hydropower plants without reservoirs (or so small that they do not count for a long enough period to be of interest here).

In order to make the analysis simple we will assume that each technology we will include can be modeled as an aggregate sector by using a cost function that reflects a unique merit order of individual generators. Specifying a conventional thermal sector (super index *C*) (including gas and coal) and a nuclear sector (super index *N*) we have sector cost functions with the output level as the argument, $c^C(e_t^C)$ and $c^N(e_t^N)$ depending on the cost of the variable primary energy source. We assume primary energy prices to be fixed for all periods and no technological change. Costs not depending on output are excluded; as such costs should not influence the current utilization.

The intermittent technologies can be lumped together because qualitatively the different technologies will be treated in the same way, with current production given by

$$e_t^I = \beta_t \bar{e}^I, \beta_t \in [0, \beta^{\max}] \qquad (19)$$

Here $\bar{e}^I$ is the capacity in power units (kW), and $\beta_t$ is the current rate of flow of the intermittent energy source (water, wind, sunshine) calculated as the average factor over the period in question converting capacity into energy (kWh) for the period.[1] We assume that there are no variable costs running the intermittent technologies, only production-independent costs.

The social planning problem can then be set up like it is done in Equation (6) with thermal costs subtracted from the gross area under the demand functions (calculated in money). We will illustrate possible optimal solutions using two consecutive periods called period 1 and 2 for ease. But remember that the illustration can be regarded as a two-period window of a general *T* period solution.

In Figure 4, we have placed a "bathtub" showing the hydropower resources for the two periods

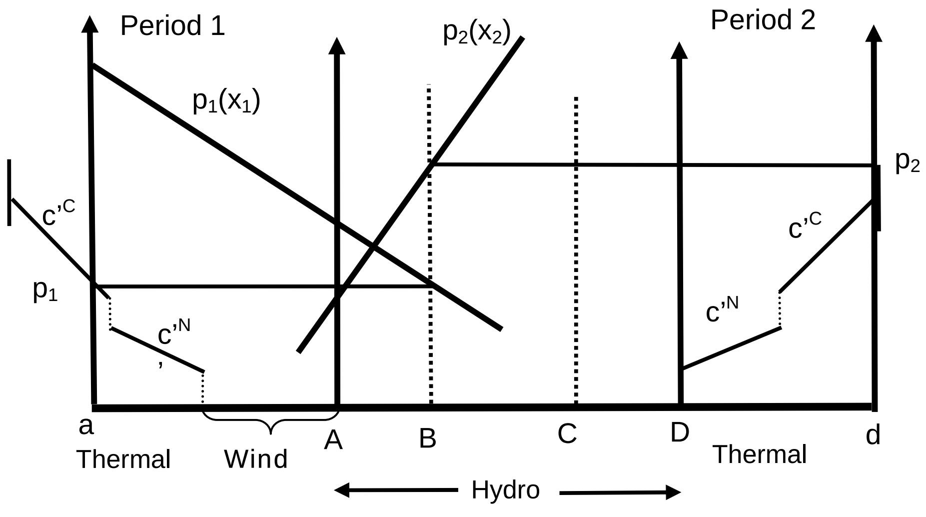


*Figure 4. Extended energy bathtub for hydropower, thermal power and intermittent power*

[1] The situation for wind power may be slightly more involved because there is a minimum wind flow necessary to start generating electricity, and the windmill has to be shut down if the wind exceeds a maximum strength and thus nothing will be produced.

in the middle of the diagram indicated by the bottom line from A to D, and by walls erected from these points. Period 1 is measured along the left-hand wall of the bathtub, and period 2 along the right-hand wall. The water resource available for period 1, made up of water inherited from the period before period 1 in the general case and the inflow during period 1, is AC, and the inflow in period 2 is CD. The storage capacity for water is given by BC, and the walls erected from these two points illustrate the reservoir capacity. For period 1, the production possibilities are extended to the left of the wall of the hydro bathtub, indicated with marginal cost curves for intermittent energy (called wind for ease), following the floor of the extended bathtub since the variable cost is zero, anchored at the left-hand hydro bathtub wall, and then comes, in merit order, the marginal cost curve for nuclear capacities ($c'^N$) and the marginal cost curve for conventional thermal capacities ($c'^C$). The length of the curves indicates the given capacity limits. The cost curves are for simplicity made linear (they could be made as step curves, as is common in applied studies). The marginal cost curves exhibit standard increasing marginal cost. There is a jump from the most expensive nuclear capacity to the cheapest conventional thermal capacity. The extension of the hydro bathtub for period 2 on the right-hand side is a mirror image of the thermal marginal cost curves for period 1, but by assumption there is no intermittent energy available.

The demand curve for electricity for period 1 is anchored on the left-hand energy wall erected from point A, and electricity consumption is measured from left to right. The demand curve for period 2 is anchored on the right-hand energy wall (the anchoring is not shown explicitly) erected from point D and electricity consumption is measured from right to left. Both demand curves are drawn linear for ease of illustration. Period 1 is a low-demand period while period 2 is a high-demand period.

The optimal solution to the management problem implies that the placement of the outer walls of the extended energy bathtub is *endogenously* determined. We erect the two walls such that we get illustrations consistent with an optimal underlying model solution of a nature we want to discuss. The equilibrium price for period 1 implies use of all three technologies in period 1. Conventional thermal is the marginal technology in the sense that total capacity is partially utilized and nuclear and wind are fully utilized. The hydro contribution is the amount of water, AB, locked in to be used in period 1 and a full reservoir BC is left for period 2. The price is equal to the marginal cost of the partially utilized conventional thermal capacity.

In period 2 without intermittent energy the situation is such that thermal capacity is fully utilized. The full water reservoir from period 1, BC, plus the inflow in period 2, CD, hence BD, is used in period 2. The result for prices is that the high-demand period has a substantial higher price than the low-demand period, and the price is higher than the marginal cost of conventional thermal due to full capacity utilization. Both hydro and thermal are used as peak load capacity.

A typical optimal solution in the pure hydro case will be that the price is the same in both periods (see Figure 2). This may still be a typical situation, and is illustrated in Figure 5. To study the

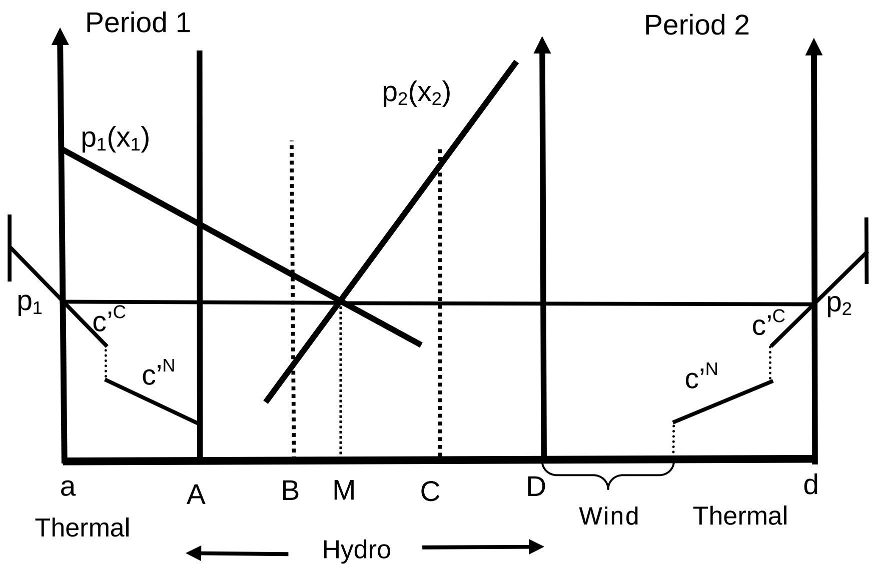


*Figure 5. Intermittent energy available only in period 2*

effect of varying wind resource, we have in Figure 5 assumed no intermittent resource on stream in period 1, but a maximal availability in period 2. The illustration shows that enough generating capacity is available in period 2 to equalize prices within the capacity limit of the conventional thermal capacity. Enough water, MC, is transferred to period 2 to keep the same price and to benefit from the wind resource. An obvious consequence of equal price is that the same amount of the partially utilized technology will be used in each period. Water and wind take care of the peak load in period 2.

The model framework illustrated above is well suited to analyze the consequences of introducing a substantial amount of intermittent power, mainly wind power, into a system rich on hydropower. In a Northern European context the hydropower of Norway is likened to an electric battery that could supply Europe when the wind is not blowing.  Opening up for trade between countries, as seen from Figure 4 Norway may import the wind power used there and save maximally on the use of hydro in that period, and then export hydroelectricity when more power is needed in Europe (Denmark and Germany) due to low wind. It should make economic sense for Norway to import cheap and export to high prices, but this also benefits the wind power countries because without trade the situation would be much worse.

It may be mentioned that a large-scale introduction of pumped storage in a hydro country like Norway will increase the inflows and thereby facilitate a larger beneficial trade with wind-rich countries. The economic rationale for pumped storage is that the difference between electricity prices with abundant wind and very little is greater than the value of the loss of pumping up water. The energy loss is in the range of 70 to 85 %.

## 4. Additional hydropower issues

*Uncertainty*

A very basic feature of hydropower operation is that inflows to the reservoirs are stochastic variables. The problems this creates for hydropower management are quite obvious. A decision about use of water, i.e., production in the current period and transferring water to the next period, has to be made in the current period while the inflows of the future periods up to the horizon are known only by their predictions. The best we can do in the current period is to formulate an optimal plan by maximising the *expectation* of the sum of consumer plus producer surpluses for all future periods.

The problem for finding optimal solutions of the hydro management problem created by uncertainty was recognised early in the literature In Norway a special solution strategy termed the *expected water value approach* was introduced by the Norwegian electricity regulator (the Norwegian Water Resources and Energy Directorate). In the more specialised engineering literature one can find many studies on solution algorithms and applied simulation studies. In the numerical system-wide model for Norway used by large producers a strategy for dealing with uncertainty, originating in the methods of the Norwegian electricity regulator, is implemented.

Taking uncertainty for the next period into account when making decisions for the current period when inflows are known involves bringing in expectations for the next period. As time evolves prices will differ from their expectations and this is a source of price variation independent of constraints in the hydro system becoming active.

*Market power*

The deregulation of the electricity power production system in many countries since the early 1990s has stimulated interest in the possibilities of producers behaving strategically. The classical implication of use of market power that production is reduced compared with perfect competition also holds for electricity markets being supplied by conventional thermal power. Systems with a significant contribution from hydro power with storage of water have not been studied so much.

The almost costless instantaneous change in hydro generation within the power capacities makes it perfect for strategic actions in competition with thermal generators, with both costs and time lags involved in changing production levels of the latter. In countries with day-ahead spot markets hydro producers interact daily and they all know that operating output-depending costs are zero, the opportunity cost is represented by future expected market prices, and they may hold quite similar expectations. This may facilitate collusion.

In the case of hydropower, production can be reduced only by using less water. However, spilling may easily be detected. One reason for concern about potential market power abuse of hydro producers is that it may be used without any spilling of water and not so easy to detect by regulators, because market power is typically exercised by a *reallocation* of release of water between periods based on differences of demand elasticities.

Interesting game situations may be set up between a limited number of actors with different technologies.

*Balancing power*

The almost instantaneous possibilities of regulating the production of electricity from hydro generators make hydropower very cost-attractive for balancing purposes. The need of balancing arises from the necessary time lag between planning the production and the actual consumption. Demand may be stochastic due to the influence of outdoor temperature, and there may be stochastic events on the producer side like accidents within generators or mishaps on the transmission lines, like lines falling out or transformers, etc. malfunctioning. Because the physical balance between supply and demand must be continuous access to regulating supply up or down is essential if short-term physical restriction of demand is to be avoided.

One issue is the pricing principles of hydropower serving balancing needs. Applying the arbitrage principle up-regulation imposes a change in the producer's optimal plan and the social cost is the difference between the spot price at the moment of regulation and the highest price obtainable in the future considering the storage capacity of the reservoir. For down-regulation if the producer has spare storage capacity the social cost is zero, but with a full reservoir the social cost is the spot price.

It may be thought that the introduction of more intermittent energy would increase the profitability of hydro in the balancing market, but this is not certain.

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